

LOCAL SYMMETRIES OF FINITE TYPE HYPERSURFACES IN \mathbb{C}^2

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ABSTRACT. The first part of this paper gives a complete description of local automorphism groups for Levi degenerate hypersurfaces of finite type in \mathbb{C}^2 . We also prove that, with the exception of hypersurfaces of the form $v = |z|^k$, local automorphisms are always determined by their 1-jets. Using this result, in the second part we describe special normal forms which by an additional normalization eliminate the nonlinear symmetries of the model and allow to decide effectively about local equivalence of two hypersurfaces given in this normal form.

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1. INTRODUCTION

The main aim of this paper is to give a complete description of local automorphism groups for Levi degenerate hypersurfaces of finite type in complex dimension two. The results rely mainly on the construction of normal forms given in [18].

The problem of describing local symmetries for real hypersurfaces in two dimensional complex space is closely related to the local equivalence problem. This connection appears already in the foundational work of H. Poincaré [21]. A natural approach to the problem, whose germs can be also found in [21], is to analyze directly the action of the group of local biholomorphic transformations at the given point. For Levi nondegenerate hypersurfaces this analysis was completed in the beautiful construction of S. S. Chern and J. Moser [7].

In recent years the same approach was applied to various classes of Levi degenerate hypersurfaces (see [3, 6, 9, 12, 15, 22, 24]). In [18], normal forms are constructed for general finite type hypersurfaces in dimension two. Since the normal forms are on the level of formal power series (neither convergence nor divergence has been proved), in order to solve Poincaré's local equivalence problem one has to combine the construction with the result of M. S. Baouendi, P. Ebenfelt and L. P. Rothschild [1] on convergence of formal equivalences between finite type hypersurfaces. In fact, there are three different normal forms defined

in [18], depending on the form of the model (see Section 2 for more details).

For Levi nondegenerate hypersurfaces, Chern-Moser's construction of normal forms, in general dimension, gives already substantial information about local symmetries, but not complete information. It shows that the local automorphism group of any hypersurface is a subgroup of the group of local symmetries of the model hyperquadric. It also implies that local automorphisms are determined by their 2-jets.

These results are strengthened and completed in an important way by the theorems of V. K. Beloshapka, N. G. Kruzhlin and A. V. Loboda ([5, 19, 23]). Local automorphisms are in fact determined by their 1-jets, whenever the hypersurface is different from the model hyperquadric. Moreover, in the strongly pseudoconvex case there exist local holomorphic coordinates in which all automorphisms are linear. Results of V. V. Ezhov ([14]) show that the last property no longer holds in general for hypersurfaces with mixed signature.

The group of local automorphisms of a hypersurface (i.e. automorphisms which fix the given point) is usually called the stability group. The problems of finite jet determination and estimation of the dimension of the stability group on Levi degenerate hypersurfaces have been intensively studied in the last decade (see [13, 11, 12, 10, 26], and the survey article [2] for further references). In dimension two, one of the most important results states that uniform finite determination, which holds on finite type hypersurfaces, actually fails for points of infinite type (see [20, 26]). More precisely, for any integer k there is an infinite type, non Levi flat hypersurface whose local automorphisms are not determined by their k -jets. On the other hand, D. Zaitsev formulated recently a conjecture that in dimension two, jets of order higher than one are needed only for hypersurfaces which are biholomorphic to the ball at generic points. Proposition 3.1 below confirms this conjecture in the finite type case.

In Section 2 we introduce notation and review the needed ingredients of the normal form construction from [18]. In Section 3 we consider hypersurfaces whose models are the higher type analogs of spheres, given by $v = |z|^k$. We prove that, except for the model hypersurfaces themselves, local automorphisms are always determined by 1-jets, and are linear in normal coordinates (it should be stressed that since the convergence of the normal forms has not been proved, these normal coordinates are a priori only formal). In Section 4 we use this result to give a complete classification of local automorphism groups for Levi degenerate hypersurfaces of finite type. The remaining analysis for non-spherical models is straightforward, as the symmetries of such models

are themselves linear. As one consequence, the result gives a complete description of hypersurfaces with finite stability group. In Section 5 we define special normal forms for hypersurfaces whose models are given by $v = |z|^k$, which allow to decide effectively about local equivalence of two hypersurfaces put into this normal form.

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During the work on this paper I learned from M. Eastwood about the work of Valerii Beloshapka and Vladimir Ezhov [6]. In this paper normal forms similar to those of [18] are described (six different cases are considered).

2. NOTATION AND PRELIMINARIES

We will consider a real analytic hypersurface $M \subseteq \mathbb{C}^2$ in a neighborhood of a point $p \in M$ in which the Levi form degenerates. The point will be assumed to be of finite type k in the sense of Kohn ([17]).

For local description of M near p we will use local holomorphic coordinates (z, w) , where $z = x + iy$, $w = u + iv$, such that $p = 0$ and the hyperplane $\{v = 0\}$ is tangent to M at p . M is then described near p as the graph of a uniquely determined real valued function

$$(1) \quad v = F(x, y, u).$$

We will consider the Taylor expansion of F expressed in terms of (z, \bar{z}, u) :

$$(2) \quad F(z, \bar{z}, u) = \sum_{i+j+m \geq 2} a_{ijm} z^i \bar{z}^j u^m,$$

where $a_{ijm} = \overline{a_{jim}}$.

Further we denote

$$(3) \quad Z_{ij}(u) = \sum_m a_{ijm} u^m,$$

hence

$$(4) \quad F(z, \bar{z}, u) = \sum_{i,j} Z_{ij}(u) z^i \bar{z}^j.$$

We will analyze the effect of a holomorphic transformation

$$(5) \quad z^* = f(z, w), \quad w^* = g(z, w),$$

on the defining equation of M . Here f and g are represented by power series

$$(6) \quad f(z, w) = \sum_{i,j=0}^{\infty} f_{ij} z^i w^j, \quad g(z, w) = \sum_{i,j=0}^{\infty} g_{ij} z^i w^j.$$

We require that such a transformation preserves the form given by (1), which means that the origin has to be mapped to itself and the hyperplane $v^* = 0$ has to be tangent to M at p in the coordinates (z^*, w^*) . This will be satisfied if and only if

$$(7) \quad f = 0, \quad g = 0, \quad g_z = 0, \quad \operatorname{Im} g_w = 0 \quad \text{at} \quad z = w = 0.$$

Only such transformations will be considered in the sequel.

We denote by F^* the function describing M in coordinates (z^*, w^*) , and write

$$(8) \quad F^*(z^*, \bar{z}^*, u^*) = \sum_{i+j+m \geq 2} a_{ijm}^* (z^*)^i (\bar{z}^*)^j (u^*)^m,$$

where again $a_{ijm}^* = \overline{a_{jim}^*}$. F^* is related to F and (f, g) by the following transformation formula:

$$(9) \quad F^*(f(z, u + iF), \overline{f(z, u + iF)}, \operatorname{Re} g(u + iF)) = \operatorname{Im} g(z, u + iF),$$

where the argument of F is (z, \bar{z}, u) . This formula gives an equality of two power series in z, \bar{z}, u , and by comparing coefficients of various monomials we may obtain explicit relations between F^* and F, f, g .

A natural tool which simplifies the use of this formula is provided by weighted coordinates. We give weight one to z and \bar{z} and weight k to u and w .

Recall that $p \in M$ is a point of finite type k in the sense of Kohn if and only if there exist local holomorphic coordinates such that M is described by

$$(10) \quad v = \sum_{j=1}^{k-1} a_j z^j \bar{z}^{k-j} + o(|z|^k, u),$$

where the leading term is a nonzero real valued homogeneous polynomial of degree k , with $a_j \in \mathbb{C}$ and $a_j = \overline{a_{k-j}}$.

The model hypersurface M_D to M at p is defined using the leading homogeneous term,

$$(11) \quad M_D = \{(z, w) \in \mathbb{C}^2 \mid v = \sum_{j=1}^{k-1} a_j z^j \bar{z}^{k-j}\}.$$

In particular, when the leading term is equal to $|z|^k$, we will write

$$(12) \quad O_k = \{(z, w) \in \mathbb{C}^2 \mid v = |z|^k\}.$$

Now we recall two basic integer invariants used in the normal form construction in [18]. The first one, denoted by l , is the essential type of the model hypersurface to M at p . It can be described as the lowest index in (11) for which $a_l \neq 0$. It satisfies $1 \leq l \leq \frac{k}{2}$.

The second invariant is defined when $l < \frac{k}{2}$ as follows. Let $l = m_0 < m_1 < \dots < m_s < \frac{k}{2}$ be the indices in (11) for which $a_{m_i} \neq 0$. The invariant, which we will denote by κ , is the greatest common divisor of the numbers $k - 2m_0, k - 2m_1, \dots, k - 2m_s$.

While for $l < \frac{k}{2}$ the stability group of M_D is one dimensional, the stability group of O_k has dimension three. Its elements are of the form (\tilde{f}, \tilde{g}) , where

$$(13) \quad \tilde{f}(\delta, \mu, \theta; z, w) = \frac{\delta e^{i\theta} z}{(1 + \mu w)^{\frac{1}{l}}}, \quad \tilde{g}(\delta, \mu, \theta; z, w) = \frac{\delta^k w}{1 + \mu w},$$

with $\delta > 0$, and $\theta, \mu \in \mathbb{R}$. We will use their Taylor expansion

$$(14) \quad \tilde{f}(\delta, \mu, \theta; z, w) = \delta e^{i\theta} \left(z - \frac{\mu}{l} z w + \dots \right)$$

and

$$(15) \quad \tilde{g}(\delta, \mu, \theta; z, w) = \delta^k (w - \mu w^2 + \dots).$$

In [18], Proposition 4.2, we proved that if the model hypersurface to M at p is O_k , there exists a unique formal transformation satisfying normalization conditions (7) and

$$(16) \quad f_z = Re \, g_w = 1, \quad Re \, g_{ww} = 0 \quad \text{at } z = w = 0,$$

which takes the defining equation for M into normal form, where the normal form conditions are

$$\begin{aligned} Z_{j0} &= 0, & j &= 0, 1, \dots, \\ Z_{l,l+j} &= 0, & j &= 0, 1, \dots, \\ Z_{2l,2l} &= 0, \\ Z_{3l,3l} &= 0, \\ Z_{2l,2l-1} &= 0. \end{aligned}$$

3. LINEARITY OF LOCAL AUTOMORPHISMS

In this section we prove a result analogous to the result of [5, 19] for nondegenerate hypersurfaces. The stability group of M at p will be denoted by H_M . In [18] we proved that if $l < \frac{k}{2}$, then $\dim H_M \leq 1$,

and all local automorphisms are determined by their 1-jets. Here we consider the case when $l = \frac{k}{2}$.

Proposition 3.1. *If M is not equivalent to O_k , then $\dim H_M \leq 1$. Moreover, all local automorphisms expressed in normal coordinates are linear.*

proof: We assume that the model is O_k , but M is not equivalent to O_k . Let us consider normal coordinates for M and separate the first two leading terms in the Taylor expansion of F ,

$$(17) \quad F(z, \bar{z}, u) = |z|^k + Q(z, \bar{z}, u) + o_{wt}(p),$$

where Q is a nonzero weighted homogeneous real valued polynomial of weight $p > k$,

$$(18) \quad Q(z, \bar{z}, u) = \sum_{\alpha+\beta+k\gamma=p} a_{\alpha\beta\gamma} z^\alpha \bar{z}^\beta u^\gamma,$$

and $o_{wt}(p)$ denotes terms which are of weight greater than p . We define the index $(\alpha_0, \beta_0, \gamma_0)$ to be the smallest one in inverse lexicographic ordering (the last components are compared first, then the second ones) for which $a_{\alpha_0, \beta_0, \gamma_0} \neq 0$.

Let (f, g) be a local automorphism of M , i.e. a transformation which preserves F . Its general form is

$$(19) \quad \begin{aligned} f(z, w) &= \delta e^{i\theta} z + \text{terms of weight } \geq 2, \\ g(z, w) &= \delta^k w + \text{terms of weight } \geq k+1. \end{aligned}$$

We will call the numbers δ , θ and $\mu = \operatorname{Re} g_{ww}$ the initial data of the automorphism, and consider simultaneously M with the automorphism (f, g) and the model O_k with the automorphism (\tilde{f}, \tilde{g}) having the same initial data as (f, g) . We will use (9) to compare the coefficients of (f, g) and (\tilde{f}, \tilde{g}) (see [18] for a detailed description of the use of the transformation rule (9)).

First we will show that f and g may be replaced by \tilde{f} and \tilde{g} when considering terms of weight less or equal to $p+k$ in (9). More precisely,

$$(20) \quad f(z, w) = \tilde{f}(z, w) + o_{wt}(p+1), \quad g(z, w) = \tilde{g}(z, w) + o_{wt}(p+k).$$

This is done in two steps. First, since Q has weight p , all equations obtained from (9) for coefficients of monomials up to weight $p-1$ are the same as those for O_k and (\tilde{f}, \tilde{g}) . Hence f is equal to \tilde{f} modulo $o_{wt}(p-k)$ and \tilde{g} equal to g modulo $o_{wt}(p-1)$. For terms of weight p , Q enters (9) only via the linear part of (f, g) , as $Q(\delta e^{i\theta} z, \delta e^{-i\theta} \bar{z}, \delta^k u)$. Since Q (and in particular $a_{\alpha_0, \beta_0, \gamma_0}$) has to be preserved, we obtain immediately that

$$\delta = 1, \quad e^{i(\alpha_0 - \beta_0)\theta} = 1.$$

For terms of weight $p+1, p+2, \dots, p+k$, Q enters (9) only through the initial data Reg_{ww} , and the coefficients f_{20}, \dots, f_{k0} in f and g_{11}, \dots, g_{k1} in g . But we already know these coefficients to be the same as in (\tilde{f}, \tilde{g}) , namely zero (if $k > p-k$ we use an obvious step by step argument). Since by the result of [18] a local automorphism is uniquely determined by its initial data, it follows that \tilde{f} has to agree with f modulo terms of weight greater than $p+1$ and \tilde{g} has to agree with g modulo terms of weight greater than $p+k$. This proves the claim.

Now we consider all terms of weight $k+1, \dots, k+p$ in the transformation formula (9). On the right hand side, using $g(z, w) = w - \mu w^2 + \dots$ we have

$$(21) \quad \begin{aligned} Im \, g(z, u + iF) &= F - Im \, \mu(u + i(|z|^k + Q + o_{wt}(p)))^2 + J_1 + \\ &+ o_{wt}(k+p) = F + 2\mu u|z|^k - 2\mu uQ + J_1 + o_{wt}(k+p), \end{aligned}$$

where J_1 denotes terms of weight $\leq k+p$ which come only from $|z|^k$, in other words terms which appear in the corresponding expansion for O_k and (\tilde{f}, \tilde{g}) (which we will not need to write down explicitly). On the left, we get from the leading term

$$(22) \quad |f(z, u + i(|z|^k + Q + o_{wt}(p)))|^k = |z - \frac{\mu}{l}z(u + i(|z|^k + Q + o_{wt}(p))) + o_{wt}(2k)|^k$$

which gives

$$(23) \quad |z|^k - 2\mu Im \, |z|^k Q + J_2 + o_{wt}(p+k) = |z|^k + J_2 + o_{wt}(p+k),$$

where again J_2 denotes all terms of weight $\leq k+p$ which come only from $|z|^k$. From the second term in $F = F^*$ we get

$$(24) \quad \begin{aligned} Q(f, \bar{f}, Re \, g) &= Q(e^{i\theta}z - e^{i\theta}\frac{\mu}{l}z(u + i(|z|^k + Q + o_{wt}(k+1))), \bar{}, \\ &u - Re \, \mu(u + i(|z|^k + o_{wt}(k)))^2 + o_{wt}(2k)). \end{aligned}$$

By the same argument as we used before for Q , since f_{20}, \dots, f_{k0} and g_{11}, \dots, g_{k1} vanish, terms of weight greater than p and less or equal to $p+k$ in F^* influence (9) only via the linear part of (f, g) . Multiplying out and taking into account that terms coming only from $|z|^k$ have to eliminate each other, we calculate the coefficients of $z^{\alpha_0} \bar{z}^{\beta_0} u^{\gamma_0+1}$ in (9). We get

$$(25) \quad a_{\alpha_0, \beta_0, \gamma_0+1} - a_{\alpha_0, \beta_0, \gamma_0} \left(\frac{1}{l} \mu (\alpha_0 + \beta_0 + l\gamma_0) \right) = -2\mu a_{\alpha_0, \beta_0, \gamma_0} + a_{\alpha_0, \beta_0, \gamma_0+1}.$$

It will hold if and only if

$$(26) \quad a_{\alpha_0, \beta_0, \gamma_0} \left(2\mu - \frac{1}{l} \mu (\alpha_0 + \beta_0 + l\gamma_0) \right) = 0,$$

hence

$$(27) \quad \alpha_0 + \beta_0 + l\gamma_0 = k$$

(recall that $l = \frac{k}{2}$). It follows that either $\mu = 0$, or $\gamma_0 = 1$ and $\alpha_0 + \beta_0 = l$. If $\gamma_0 = 1$ we consider the coefficients of $z^{\alpha_0+k} \bar{z}^{\beta_0+k}$. From the formulas above we get

$$(28) \quad a_{\alpha_0+k, \beta_0+k, 0} + \mu a_{\alpha_0, \beta_0, 1} = a_{\alpha_0+k, \beta_0+k, 0},$$

and so $\mu = 0$. Hence there is no automorphism of M with $\mu \neq 0$, and we proved that every local automorphism in normal coordinates is linear. To prove that $\dim H_M \leq 1$, it is enough to realize that linear automorphisms act on each term in F individually, and that dilations preserve only the homogeneous model.

4. CLASSIFICATION OF LOCAL SYMMETRIES

The result of Proposition 3.1 can be used to obtain a complete classification of local automorphism groups.

Proposition 4.1. *For a given hypersurface M exactly one of the following possibilities occurs:*

- (1) H_M has real dimension three. This happens if and only if M is equivalent to O_k .
- (2) H_M is isomorphic to $\mathbb{R}^+ \oplus \mathbb{Z}_m$. This happens if and only if M is a model hypersurface with $l < \frac{k}{2}$, and $m = \kappa$ when k is even or $m = 2\kappa$ when k is odd.
- (3) H_M is isomorphic to S^1 . This happens if and only if M is weakly spherical, i.e. the defining equation in normal coordinates has form

$$v = G(|z|^2, u).$$

- (4) H_M is finite, isomorphic to \mathbb{Z}_n for some $n \in \mathbb{N}$.

Note that the last case includes the trivial symmetry group.

proof: By Proposition 3.1, if $l = \frac{k}{2}$ and M is not equivalent to O_k , then the only transformations which may preserve it in normal coordinates are the decoupled linear transformations

$$(29) \quad w^* = \delta^k w, \quad z^* = \delta e^{i\theta} z,$$

which act on each term in the expansion of F individually in an obvious way. Each such transformation can be uniquely factored into

the composition of a rotation in the z variable and a weighted dilation. If the rotation $z^* = e^{i\theta}z$ preserves all terms in F for every θ , then each must have form $a_{m_1, m_1, m_2}|z|^{2m_1}u^{m_2}$, corresponding to the third case. Further, if for one particular θ the rotation preserves a term $a_{\alpha\beta\gamma}z^\alpha\bar{z}^\beta u^\gamma = |z|^{2\alpha} \operatorname{Re} a_{\alpha\beta\gamma} z^{\beta-\alpha} u^\gamma$, where $\alpha \leq \beta$, then $e^{i\theta}$ is a $(\beta - \alpha) - th$ root of unity, and we are in cases (2) and (4). On the other hand, in all cases, a weighted dilation can preserve only terms which are weighted homogeneous of weight k . In this case M has to be a model. Further, for $l < \frac{k}{2}$ it follows from [18] that H_M is a subgroup of $\mathbb{R}^+ \oplus \mathbb{Z}_m$, with the claimed relation between m and κ . Hence, if M is not a model, H_M has to be a subgroup of \mathbb{Z}_m , i.e. it is isomorphic to \mathbb{Z}_n for some $n \in \mathbb{N}$.

5. SPECIAL NORMAL FORMS

In this part we use the calculations from Section 2 to obtain a normal form which can be used effectively to decide about local equivalence of two hypersurfaces given in normal form. The main difficulty in applying Chern-Moser's normal form for that purpose is not the dimension of the symmetry group, but rather the fact that the group does not act on the defining equation directly. Application of an element of the group can lead to an equation not in normal form. To obtain the group action on normal forms one has to perform the transformation back into normal form. The same situation occurs for the normal forms in [18], in the case when $l = \frac{k}{2}$ and the model is O_k .

We will speak about a special normal form if it practically allows to decide about equivalence or non-equivalence of two hypersurfaces which are put into this normal form. More precisely, in the two dimensional case this means that only the explicit action of a (decoupled) linear transformation is to be considered. In the nondegenerate case such a normal form is described in [7] for non-umbilical points in \mathbb{C}^2 and in [24] for higher dimensions.

We rewrite Q in the form

$$Q(z, \bar{z}, u) = \sum_{\alpha+\beta+k\gamma=p} |z|^{2\alpha} \operatorname{Re} a_{\alpha\beta\gamma} z^{\beta-\alpha} u^\gamma.$$

where the sum is taken over multiindices with $\alpha \leq \beta$. Recall that $(\alpha_0, \beta_0, \gamma_0)$ is the smallest index in (11) in inverse lexicographic ordering for which $a_{\alpha_0, \beta_0, \gamma_0}$ is different from zero.

First we normalize the linear part of a transformation into normal form by requiring that

$$a_{\alpha_0, \beta_0, \gamma_0} = 1.$$

This condition provides a partial normalization or a complete one, depending on the form of Q . In all cases it normalizes fully the dilation part, while θ is left free if Q consists of a single term of the form $|z|^m u^s$. In the second step we normalize the nonlinear part. As in the proof of Proposition 3.1 we have to consider two cases. If

$$(30) \quad \alpha_0 + \beta_0 + l\gamma_0 \neq k,$$

we normalize by asking that

$$(31) \quad \operatorname{Re} a_{\alpha_0, \beta_0, \gamma_0+1} = 0.$$

In the second case, when $\gamma_0 = 1$ and $\alpha + \beta = l$ we normalize by requiring that

$$(32) \quad \operatorname{Re} a_{\alpha_0+k, \beta_0+k, 0} = 0.$$

By the calculation in the proof of Proposition 3.1, these conditions determines uniquely the parameter μ .

Thus verifying local equivalence of two such hypersurfaces is reduced to the straightforward action of linear transformations.

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